Resummation of low p_T differential distributions in Soft-collinear Effective Theory

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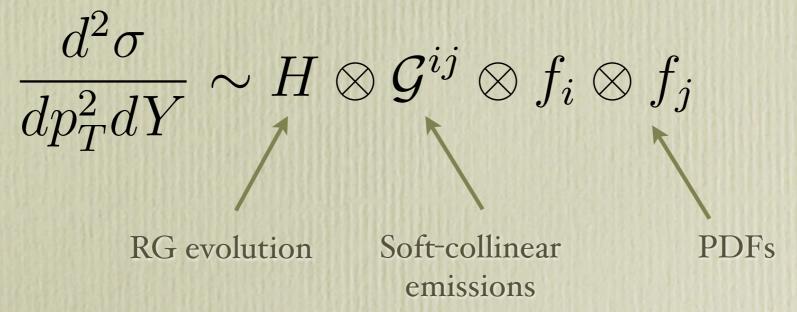
arxiv:1007.xxxx

LoopFest IX, June 22, 2010

Outline

- Introductory Remarks
- Collins-Soper-Sterman approach to low-p_T resummation
- Soft-collinear effective theory approach

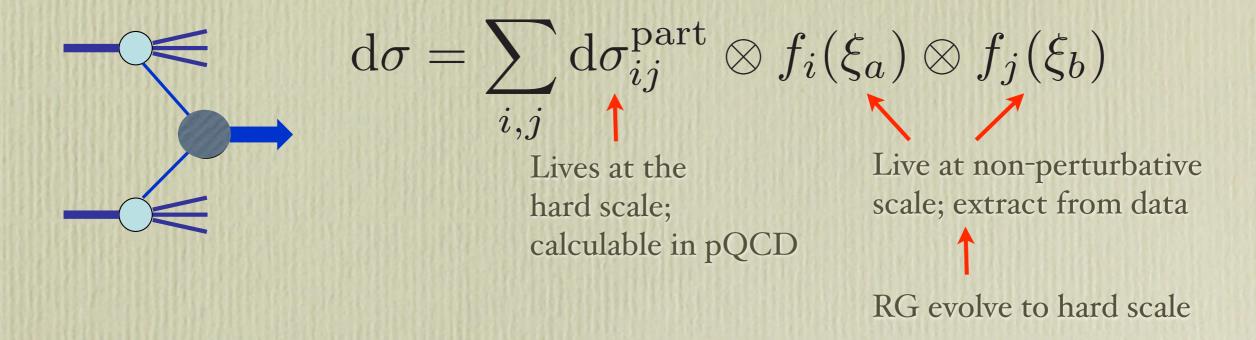
-Factorization and resummation formula:



Conclusions

Factorization and Resummation

• Fully inclusive Drell-Yan, Higgs:

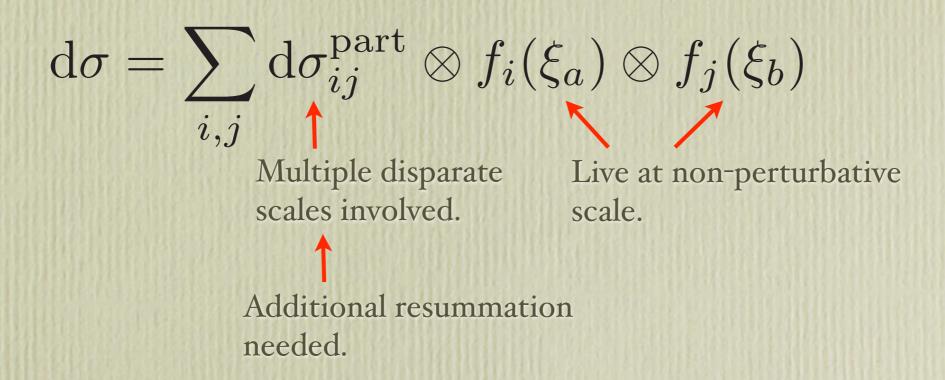


 Large logarithms of hard and non-perturbative scales arise → Resummation needed

• Resummation done by evaluating PDFs at the hard scale after renormalization group running (DGLAP)

Resummation

• In the presence of final state restrictions:



• Example: low transverse momentum distribution in Drell-Yan, Higgs production

Low pT Region

• The schematic perturbative series for the pT distribution for pp \rightarrow (h,V)+X

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2} \left[A_1 \alpha_S \ln \frac{M^2}{p_T^2} \ + \ A_2 \alpha_S^2 \ln^3 \frac{M^2}{p_T^2} \ + \ \dots + \ A_n \alpha_S^n \ln^{2n-1} \frac{M^2}{p_T^2} \ + \ \dots \right]$$

Large Logarithms spoil perturbative convergence

- Resummation of large logarithms required
- Low pt resummation has been studied in great detail

(Dokshitzer, Dyakonov, Troyan; Parisi, Petronzio; Curci et al.; Davies, Stirling; Collins, Soper, Sterman; Arnold, Kauffman; Berger, Qiu; Ellis, Ross, Veseli; Ladinsky, Yuan; Bozzi, Catani, de Florian, Grazzini,....)

• Low pt region important for W mass, Higgs searches, ...

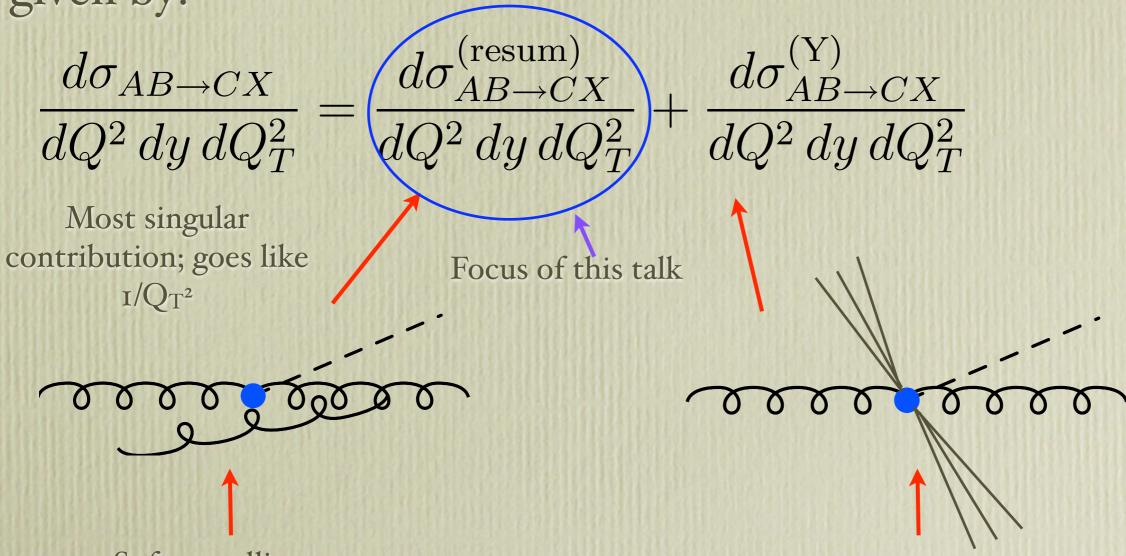
Collins-Soper-Sterman Formalism

CSS Formalism

$$A(P_A) + B(P_B) \rightarrow C(Q) + X, \quad C = \gamma^*, W^{\pm}, Z, h$$

• The transverse momentum distribution is schematically





Soft or collinear gluon emissions

Contributions from hard jets

CSS Formalism

• The CSS resummation formula takes the form:

$$\frac{d^{2}\sigma}{dp_{T} dY} = \sigma_{0} \int \frac{d^{2}b_{\perp}}{(2\pi)^{2}} e^{-i\vec{p}_{T}\cdot\vec{b}_{\perp}} \sum_{a,b} \left[C_{a} \otimes f_{a/P} \right] (x_{A}, b_{0}/b_{\perp}) \left[C_{b} \otimes f_{b/P} \right] (x_{B}, b_{0}/b_{\perp})$$

$$\times \exp \left\{ \int_{b_{0}^{2}/b_{\perp}^{2}}^{\hat{Q}^{2}} \frac{d\mu^{2}}{\mu^{2}} \left[\ln \frac{\hat{Q}^{2}}{\mu^{2}} A(\alpha_{s}(\mu^{2})) + B(\alpha_{s}(\mu^{2})) \right] \right\}.$$

Coefficients with well defined perturbative expansions

Why b-space?

• Both matrix elements and phase space simplify in softemission limit

Eikonal approximation (soft photons):

$$\mathcal{M}_n \propto g^n \mathcal{M}_0 \left\{ \frac{p_1 \cdot \epsilon_1 \dots p_1 \cdot \epsilon_n}{p_1 \cdot k_1 \dots p_1 \cdot k_n} + (-1)^n \frac{p_2 \cdot \epsilon_1 \dots p_2 \cdot \epsilon_n}{p_2 \cdot k_1 \dots p_2 \cdot k_n} \right\}$$

Phase space:

$$d\Pi_n \propto \nu(k_{T1}) d^2 k_{T1} \dots \nu(k_{Tn}) d^2 k_{Tn} \delta^{(2)} \left(\vec{p}_T - \sum_i \vec{k}_{Ti} \right)$$

$$\nu(k_T) = k_T^{-2\epsilon} \ln \left(\frac{s}{k_T^2} \right)$$

Would be independent except for phase-space constraint;
 Fourier transform to b-space accomplishes this

$$\int \frac{d^2b}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{p}_T} \int d^2k_{T1} f(k_{T1}) \dots d^2k_{Tn} f(k_{Tn}) \delta^{(2)} \left(\vec{p}_T - \sum_i \vec{k}_{Ti}\right)
= \int \frac{d^2b}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{p}_T} \left[\tilde{f}(b)\right]^n, \quad \tilde{f}(b) = \int d^2k_T e^{i\vec{b}\cdot\vec{k}_T} f(k_T)$$

CSS Formalism

$$\frac{d^{2}\sigma}{dp_{T} dY} = \sigma_{0} \int \frac{d^{2}b_{\perp}}{(2\pi)^{2}} e^{-i\vec{p}_{T}\cdot\vec{b}_{\perp}} \sum_{a,b} \left[C_{a} \otimes f_{a/P} \right] (x_{A}, b_{0}/b_{\perp}) \left[C_{b} \otimes f_{b/P} \right] (x_{B}, b_{0}/b_{\perp})$$

$$\times \exp \left\{ \int_{b_{0}^{2}/b_{\perp}^{2}}^{\hat{Q}^{2}} \frac{d\mu^{2}}{\mu^{2}} \left[\ln \frac{\hat{Q}^{2}}{\mu^{2}} A(\alpha_{s}(\mu^{2})) + B(\alpha_{s}(\mu^{2})) \right] \right\}.$$
Landau Pole

- The integration over the impact parameter introduces a Landau pole
- Must specify a treatment of the Landau pole for any value of p_T

Landau-pole prescriptions

• Introduce cutoff for the large b region by evaluating at the point (Collins, Soper 1982)

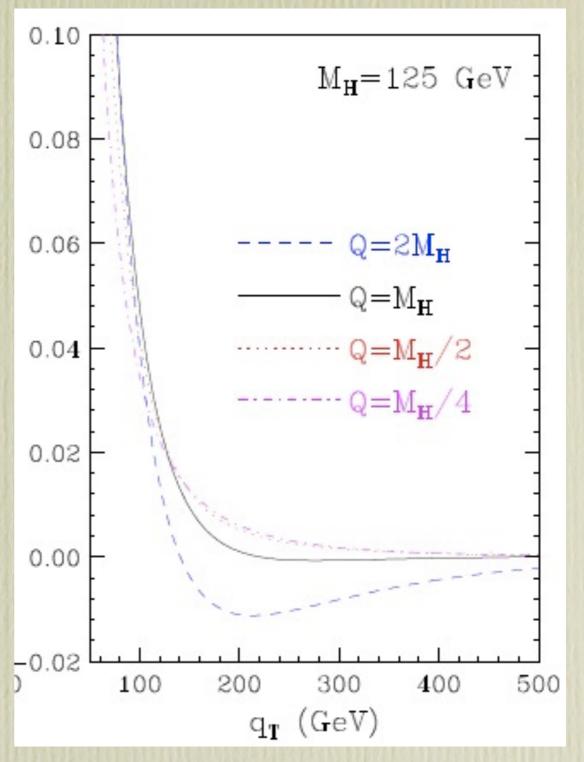
$$b_* = \frac{b}{\sqrt{1 + \left(b/b_{max}\right)^2}}$$

• "Minimal prescription:" deform b-contour to avoid singularities (Catani, Mangano, Nason, Trentadue 1996; Laenen, Sterman, Vogelsang 2000)

$$b = [\cos \phi \pm i \sin \phi] t$$

Matching to fixed-order

 Resummed exponent in bspace, fixed-order in p_T space ⇒ leads to difficulties in matching



Bozzi, Catani, de Florian, Grazzini 2005

EFT Approach

Effective Field Theory (EFT)

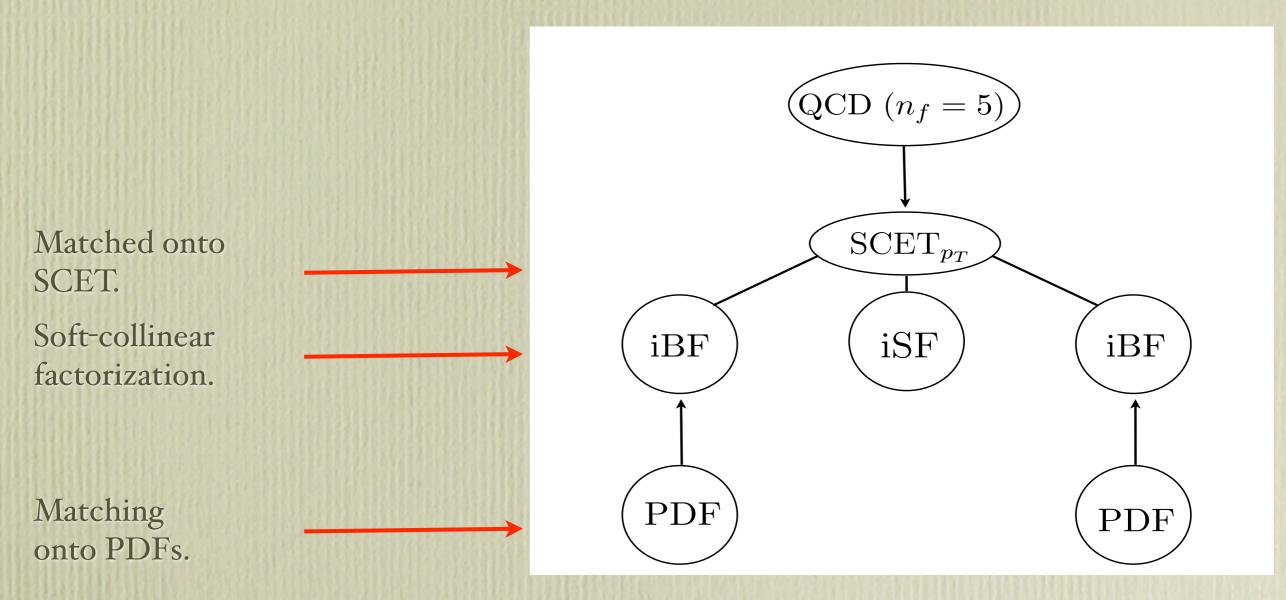
• Low transverse momentum distribution has the scales

$$m_h \gg p_T \gg \Lambda_{QCD}$$

- The most singular pT emissions are soft and collinear emissions ⇒Soft-Collinear Effective Theory (SCET) (Bauer, Fleming, Luke, Pirjol, Stewart)
- Study of SCET at the LHC, particularly for differential quantities, is still in its infancy
 - threshold resummation for inclusive Drell-Yan, Higgs, ttbar (Becher, Neubert et al.)
 - Factorization at the LHC for jet cross sections (Stewart, Tackmann, Waalewijn)
- Gain knowledge of how to apply SCET to hadronic collisions from this study

EFT framework

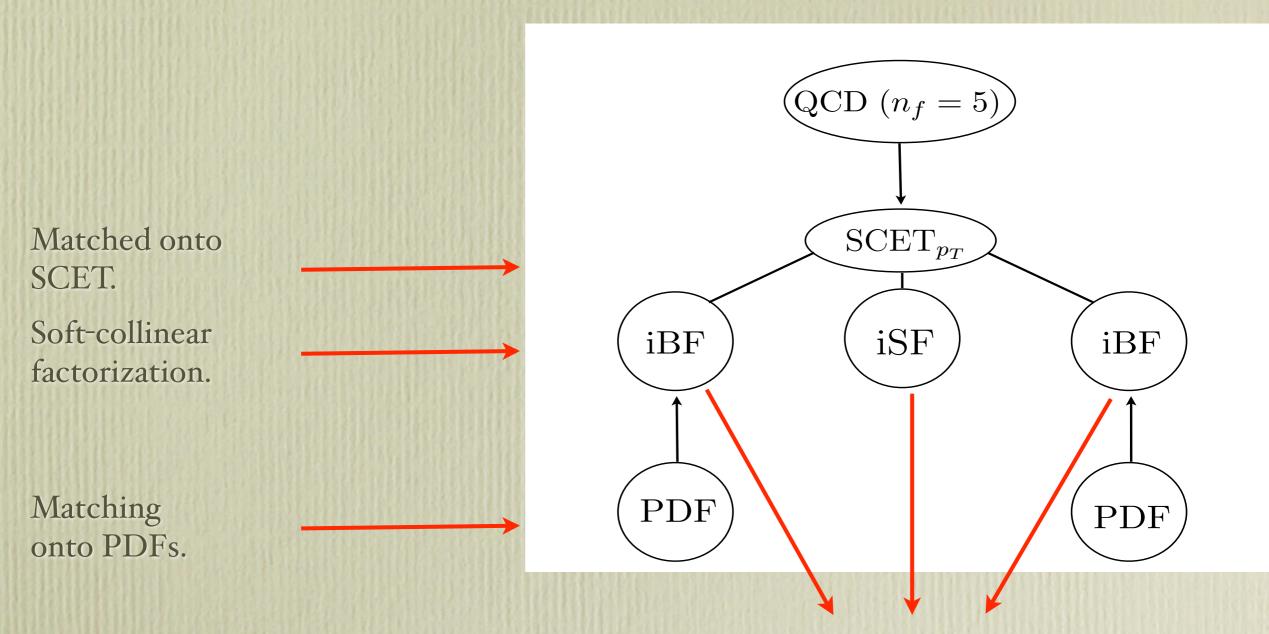
$$QCD(n_f = 6) \rightarrow QCD(n_f = 5) \rightarrow SCET_{p_T} \rightarrow SCET_{\Lambda_{QCD}}$$



Show derivation for Higgs, but identical for V=W, Z, γ^*

EFT framework

$$QCD(n_f = 6) \rightarrow QCD(n_f = 5) \rightarrow SCET_{p_T} \rightarrow SCET_{\Lambda_{QCD}}$$

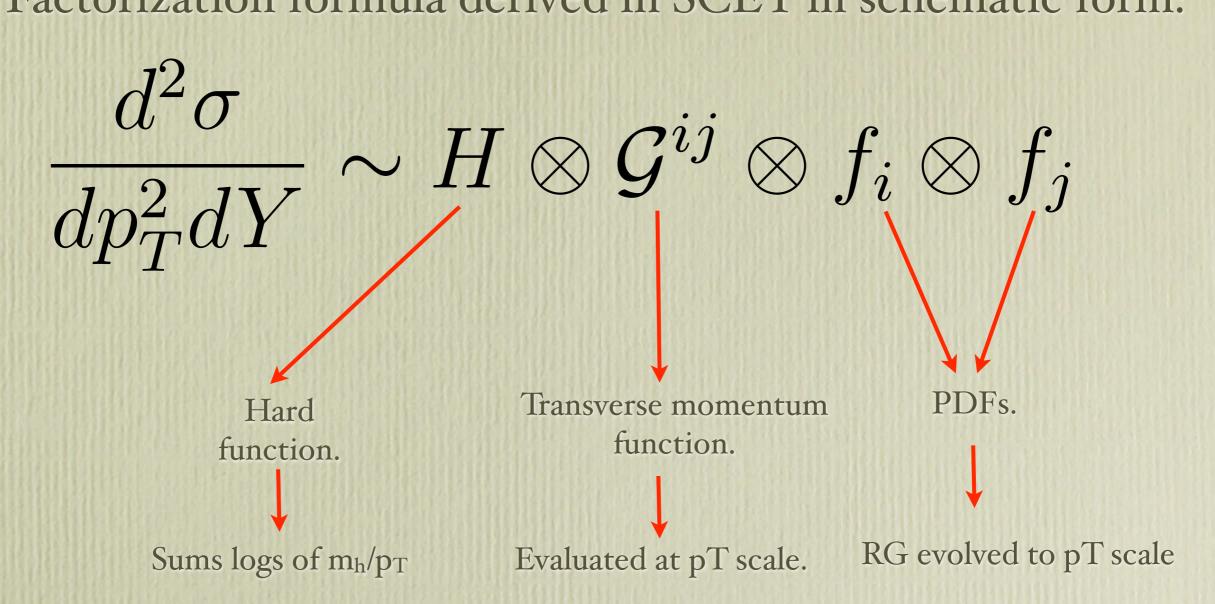


iBF = impact-parameter Beam Function
iSF = inverse Soft Function

Newly defined objects describing soft and collinear pT emissions

SCET Factorization Formula

• Factorization formula derived in SCET in schematic form:



- All objects are field theoretically defined
- Large logarithms are summed via RG equations in EFTs
- Formulation avoids Landau pole

SCET in a NutShell

• Effective theory with soft and collinear degrees of freedom:

$$p^{\mu} \equiv (p^+, p^-, p_{\perp})$$

 $p_n \sim m_h(\eta^2, 1, \eta), \quad p_{\bar{n}} \sim m_h(1, \eta^2, \eta), \quad p_s \sim m_h(\eta, \eta, \eta),$

• Well defined power counting:

$$\left(\eta \sim \frac{p_T}{m_h} \right)$$
 — Corresponds to soft and collinear modes with transverse momentum of order p_T

• Soft and collinear fields are distinguished and are decoupled at leading order in $\boldsymbol{\eta}$

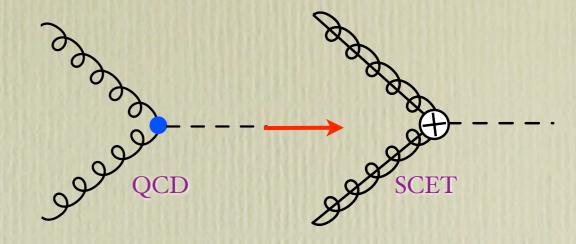
$$ig(ig(\mathcal{O}_{ ext{SCET}}ig)
ightarrow ig\langle \mathcal{O}_{soft}ig)$$

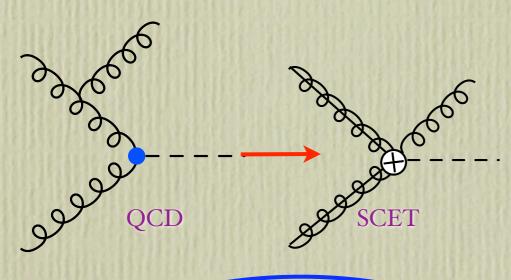
 Soft and Collinear gauge invariance restricts the form of SCET operators that can appear

Matching onto SCET

• Matching equation:

$$O_{QCD} = \int d\omega_1 \int d\omega_2 C(\omega_1, \omega_2) \mathcal{O}(\omega_1, \omega_2)$$

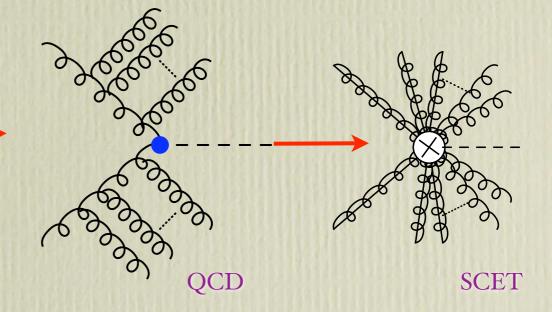




Matching real emission graphs

Tree level matching (EFT graphs scale-less in dim-reg ⇒ finite part of virtual corrections)

Soft and collinear emissions build into Wilson lines determined by soft and collinear gauge invariance



Effective SCET

operator:

$$\mathcal{O}(\omega_1, \omega_2) = g_{\mu\nu} h \, T\{ \text{Tr} \Big[S_n (gB_{n\perp}^{\mu})_{\omega_1} S_n^{\dagger} S_{\bar{n}} (gB_{\bar{n}\perp}^{\nu})_{\omega_2} S_{\bar{n}}^{\dagger} \Big] \}$$

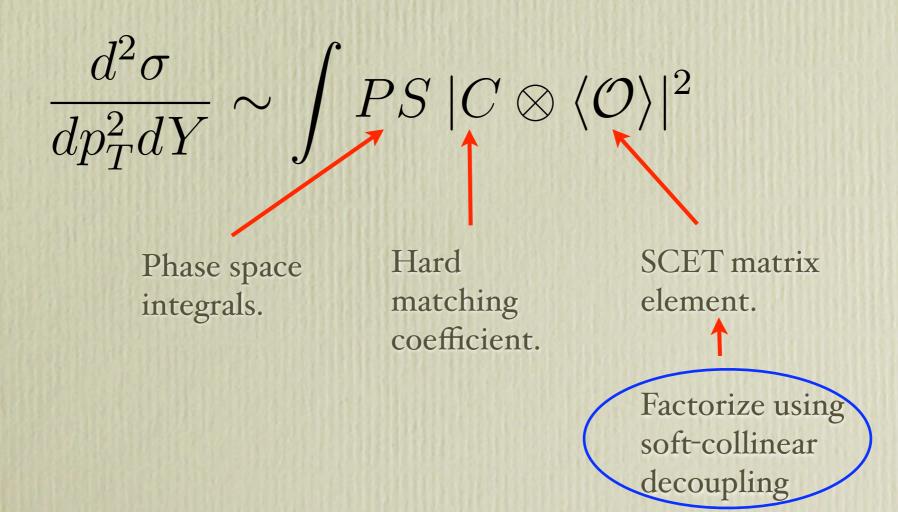
$(QCD (n_f = 5))$ (BF) (BF) (BF) (PDF) (PDF) (PDF) (PDF) (PDF) (PDF)

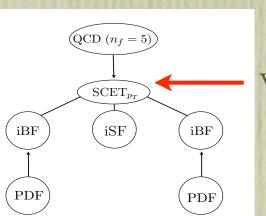
SCET Cross-Section

• SCET differential cross-section:

$$\frac{d^{2}\sigma}{du\,dt} = \frac{1}{2Q^{2}} \left[\frac{1}{4} \right] \int \frac{d^{2}p_{h_{\perp}}}{(2\pi)^{2}} \int \frac{dn \cdot p_{h}d\bar{n} \cdot p_{h}}{2(2\pi)^{2}} (2\pi)\theta(n \cdot p_{h} + \bar{n} \cdot p_{h})\delta(n \cdot p_{h}\bar{n} \cdot p_{h} - \vec{p}_{h_{\perp}}^{2} - m_{h}^{2})
\times \delta(u - (p_{2} - p_{h})^{2})\delta(t - (p_{1} - p_{h})^{2}) \sum_{\text{initial pols.}} \sum_{X} \left| C(\omega_{1}, \omega_{2}) \otimes \langle hX_{n}X_{\bar{n}}X_{s} | \mathcal{O}(\omega_{1}, \omega_{2}) | pp \rangle \right|^{2}
\times (2\pi)^{4} \delta^{(4)}(p_{1} + p_{2} - P_{X_{\bar{n}}} - P_{X_{\bar{n}}} - P_{X_{\bar{s}}} - p_{h}),$$

• Schematic form of SCET crosssection:



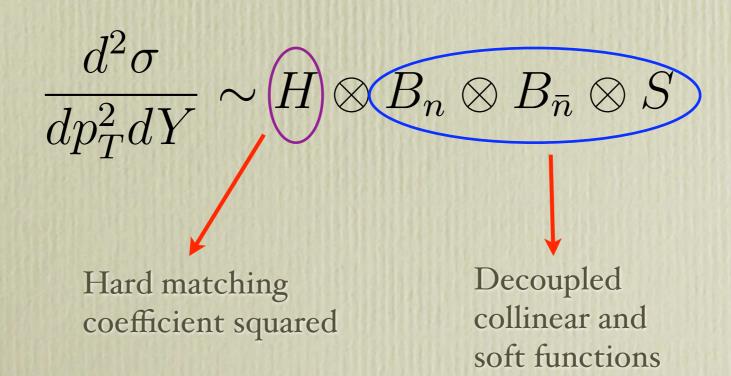


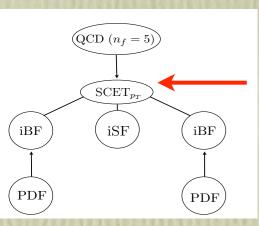
Factorization in SCET

We are here

$$\frac{d^2\sigma}{dp_T^2dY} \sim \int PS |C| \otimes \langle O \rangle|^2$$

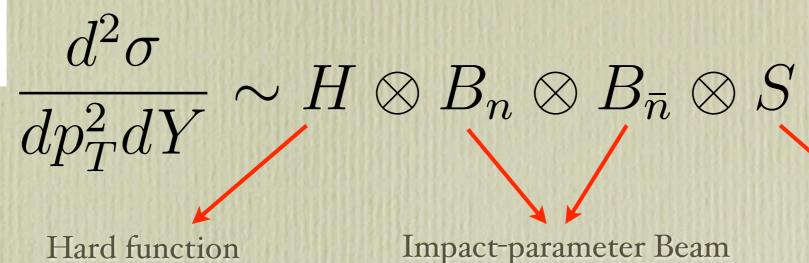
Factorize cross-section using soft-collinear decoupling in SCET



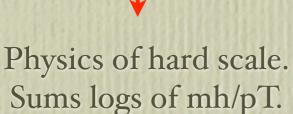


Factorization in SCET

We are here



Soft function



Describes collinear pT emissions

Functions

(iBFs)

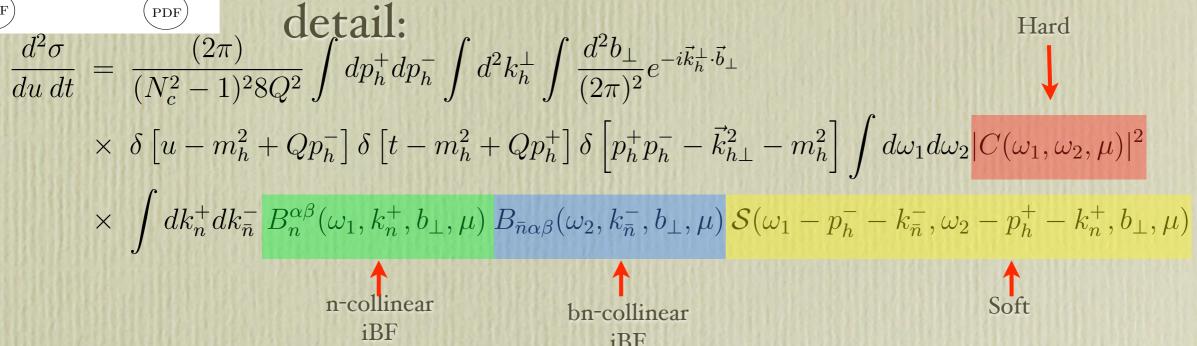
Describes soft pT emissions

Factorization in SCET

 $(QCD (n_f = 5))$ $SCET_{p_T}$ iBF iBF PDF PDF

We are here

• Factorization formula in full



• iBFs and soft functions field theoretically defined as the fourier transform of:

$$\frac{J_{n}^{\alpha\beta}(\omega_{1}, x^{-}, x_{\perp}, \mu)}{J_{\bar{n}}^{\alpha\beta}(\omega_{1}, y^{+}, y_{\perp}, \mu)} = \sum_{\text{initial pols.}} \langle p_{1} | \left[g B_{1n\perp\beta}^{A}(x^{-}, x_{\perp}) \delta(\bar{\mathcal{P}} - \omega_{1}) g B_{1n\perp\alpha}^{A}(0) \right] | p_{1} \rangle$$

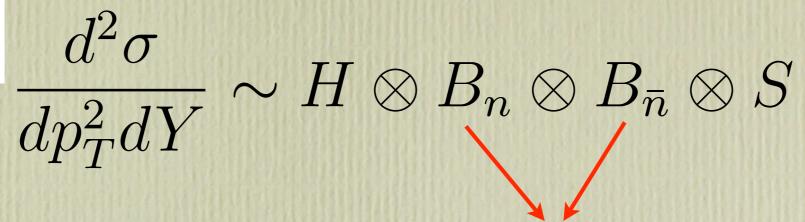
$$\frac{J_{\bar{n}}^{\alpha\beta}(\omega_{1}, y^{+}, y_{\perp}, \mu)}{J_{\bar{n}}^{\alpha\beta}(\omega_{1}, y^{+}, y_{\perp}, \mu)} = \sum_{\text{initial pols.}} \langle p_{2} | \left[g B_{1n\perp\beta}^{A}(y^{+}, y_{\perp}) \delta(\bar{\mathcal{P}} - \omega_{2}) g B_{1n\perp\alpha}^{A}(0) \right] | p_{2} \rangle$$

$$S(z, \mu) = \langle 0 | \bar{T} \left[\text{Tr} \left(S_{\bar{n}} T^{D} S_{\bar{n}}^{\dagger} S_{n} T^{C} S_{n}^{\dagger} \right) (z) \right] T \left[\text{Tr} \left(S_{n} T^{C} S_{n}^{\dagger} S_{\bar{n}} T^{D} S_{\bar{n}}^{\dagger} \right) (0) \right] | 0 \rangle$$

$(QCD (n_f = 5))$ $SCET_{p_T}$ iBF iBF PDF

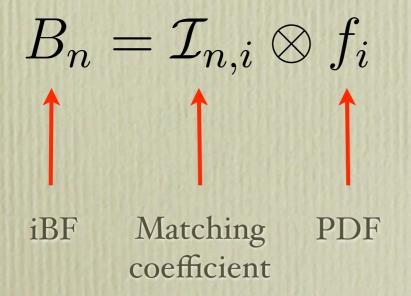
Factorization in SCET

We are here



iBFs are proton matrix elements and sensitive to the non-perturbative scale

• The iBFs are matched onto PDFs to separate the perturbative and non-perturbative scales:



$$B_{\bar{n}} = \mathcal{I}_{\bar{n},i} \otimes f_i$$

$(QCD (n_f = 5))$ $SCET_{p_T}$ IBF IBF PDF PDF

Factorization in SCET

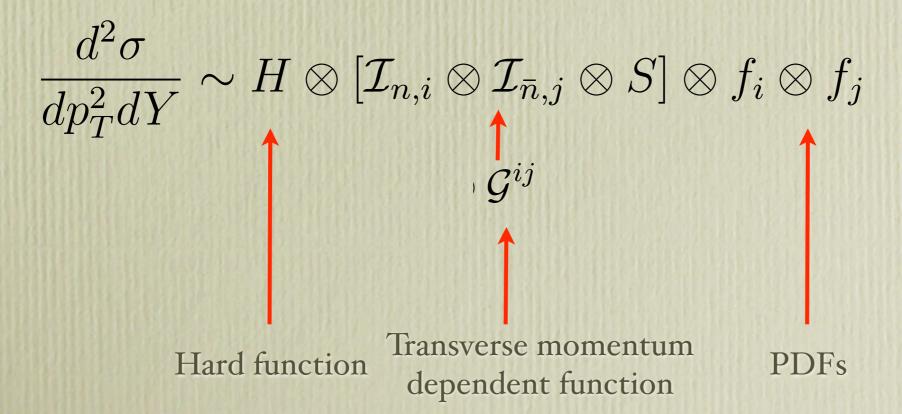
We are here

$$\frac{d^2\sigma}{dp_T^2dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

• After matching the iBFs to the PDFs we get:

$$\frac{d^2\sigma}{dp_T^2dY} \sim H \otimes [\mathcal{I}_{n,i} \otimes f_i] \otimes [\mathcal{I}_{\bar{n},j} \otimes f_j] \otimes S$$

• Group the perturbative pT scale functions into transverse momentum dependent function:



• Factorization formula in full detail:

$$\frac{d^{2}\sigma}{dp_{T}^{2} dY} = \frac{\pi^{2}}{4(N_{c}^{2} - 1)^{2}Q^{2}} \int_{0}^{1} \frac{dx_{1}}{x_{1}} \int_{0}^{1} \frac{dx_{2}}{x_{2}} \int_{x_{1}}^{1} \frac{dx'_{1}}{x'_{1}} \int_{x_{2}}^{1} \frac{dx'_{2}}{x'_{2}} \times \frac{H(x_{1}, x_{2}, \mu_{Q}; \mu_{T})\mathcal{G}^{ij}(x_{1}, x'_{1}, x_{2}, x'_{2}, p_{T}, Y, \mu_{T}) f_{i/P}(x'_{1}, \mu_{T}) f_{j/P}(x'_{2}, \mu_{T})}$$
Hard function
Transverse momentum
PDFs
function

• The transverse momentum function is a convolution of the iBF matching coefficients and the soft function:

$$\mathcal{G}^{ij}(x_{1}, x'_{1}, x_{2}, x'_{2}, p_{T}, Y, \mu_{T}) = \int dt_{n}^{+} \int dt_{\bar{n}}^{-} \int \frac{d^{2}b_{\perp}}{(2\pi)^{2}} J_{0}(|\vec{b}_{\perp}|p_{T})
\times \mathcal{I}_{n;g,i}^{\beta\alpha}(\frac{x_{1}}{x'_{1}}, t_{n}^{+}, b_{\perp}, \mu_{T}) \mathcal{I}_{\bar{n};g,j}^{\beta\alpha}(\frac{x_{2}}{x'_{2}}, t_{\bar{n}}^{-}, b_{\perp}, \mu_{T})
\times \mathcal{S}^{-1}(x_{1}Q - e^{Y} \sqrt{p_{T}^{2} + m_{h}^{2}} - \frac{t_{\bar{n}}^{-}}{Q}, x_{2}Q - e^{-Y} \sqrt{p_{T}^{2} + m_{h}^{2}} - \frac{t_{n}^{+}}{Q}, b_{\perp}, \mu_{T})$$

• Factorization formula in full detail:

$$\frac{d^{2}\sigma}{dp_{T}^{2} dY} = \frac{\pi^{2}}{4(N_{c}^{2}-1)^{2}Q^{2}} \int_{0}^{1} \frac{dx_{1}}{x_{1}} \int_{0}^{1} \frac{dx_{2}}{x_{2}} \int_{x_{1}}^{1} \frac{dx'_{1}}{x'_{1}} \int_{x_{2}}^{1} \frac{dx'_{2}}{x'_{2}} \times H(x_{1}, x_{2}, \mu_{Q}(\mu_{T}) \mathcal{G}^{ij}(x_{1}, x'_{1}, x_{2}, x'_{2}, p_{T}, Y, \mu_{T}) f_{i/P}(x'_{1}(\mu_{T}) f_{j/P}(x'_{2}, \mu_{T})) dx_{2}$$

RG evolution cut off at $\mu_T \sim p_T$, the matching scale from QCD \rightarrow SCET_{pT}, not 1/b_{\(\perp}}

Impact parameter appears, but only to simplify iBF→PDF matching; can transform this formula to be completely in momentum space

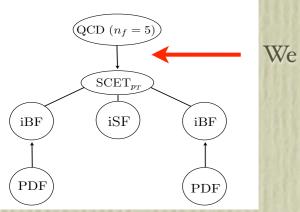
$$\mathcal{G}^{ij}(x_{1}, x'_{1}, x_{2}, x'_{2}, p_{T}, Y, \mu_{T}) = \int dt_{n}^{+} \int dt_{\bar{n}}^{-} \int \frac{d^{2}b_{\perp}}{(2\pi)^{2}} J_{0}(|\vec{b}_{\perp}|p_{T})
\times \mathcal{I}_{n;g,i}^{\beta\alpha}(\frac{x_{1}}{x'_{1}}, t_{n}^{+}, b_{\perp}, \mu_{T}) \mathcal{I}_{\bar{n};g,j}^{\beta\alpha}(\frac{x_{2}}{x'_{2}}, t_{\bar{n}}^{-}, b_{\perp}, \mu_{T})
\times \mathcal{S}^{-1}(x_{1}Q - e^{Y} \sqrt{p_{T}^{2} + m_{h}^{2}} - \frac{t_{\bar{n}}^{-}}{Q}, x_{2}Q - e^{-Y} \sqrt{p_{T}^{2} + m_{h}^{2}} - \frac{t_{n}^{+}}{Q}, b_{\perp}, \mu_{T})$$

Impact parameter appears, but only to simplify iBF→PDF matching; can transform this formula to be completely in momentum space

$$\mathcal{G}^{ij}(x_{1}, x_{1}', x_{2}, x_{2}', p_{T}, Y, \mu_{T}) = \frac{1}{2\pi} \int dt_{n}^{+} \int dt_{n}^{-} \int d^{2}k_{n}^{\perp} \int d^{2}k_{n}^{\perp} \int d^{2}k_{us}^{\perp} \frac{\delta(p_{T} - |\vec{k}_{n}^{\perp} + \vec{k}_{n}^{\perp} + \vec{k}_{us}^{\perp}|)}{p_{T}} \times \mathcal{I}_{n;g,i}^{\beta\alpha}(\frac{x_{1}}{x_{1}'}, t_{n}^{+}, k_{n}^{\perp}, \mu_{T}) \mathcal{I}_{n;g,j}^{\beta\alpha}(\frac{x_{2}}{x_{2}'}, t_{n}^{-}, k_{n}^{\perp}, \mu_{T}) \times \mathcal{S}^{-1}(x_{1}Q - e^{Y}\sqrt{p_{T}^{2} + m_{h}^{2}} - \frac{t_{n}^{-}}{Q}, x_{2}Q - e^{-Y}\sqrt{p_{T}^{2} + m_{h}^{2}} - \frac{t_{n}^{+}}{Q}, k_{us}^{\perp}, \mu_{T}) \tag{54}$$

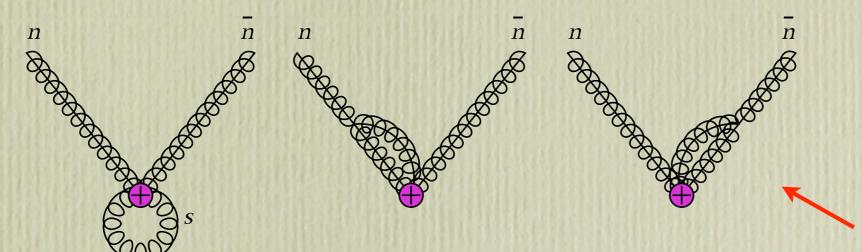
Fixed order and Matching Calculations

One loop Matching onto SCET



We are here

$$O_{QCD} = \int d\omega_1 \int d\omega_2 C(\omega_1, \omega_2) \mathcal{O}(\omega_1, \omega_2)$$



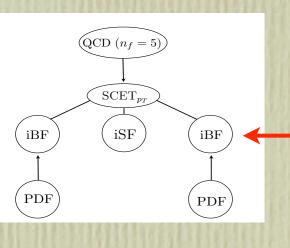
One loop SCET graphs

All graphs scaleless and vanish in dimensional regularization.

 Wilson Coefficient obtained from finite part in dimensional regularization of the QCD result for gg→h. At one loop we have:

$$C(\bar{n} \cdot \hat{p}_1 n \cdot \hat{p}_2, \mu) = \frac{c \,\bar{n} \cdot \hat{p}_1 n \cdot \hat{p}_2}{v} \left\{ 1 + \frac{\alpha_s}{4\pi} C_A \left[\frac{11}{2} + \frac{\pi^2}{6} - \ln^2 \left(-\frac{\bar{n} \cdot \hat{p}_1 n \cdot \hat{p}_2}{\mu^2} \right) \right] \right\}$$

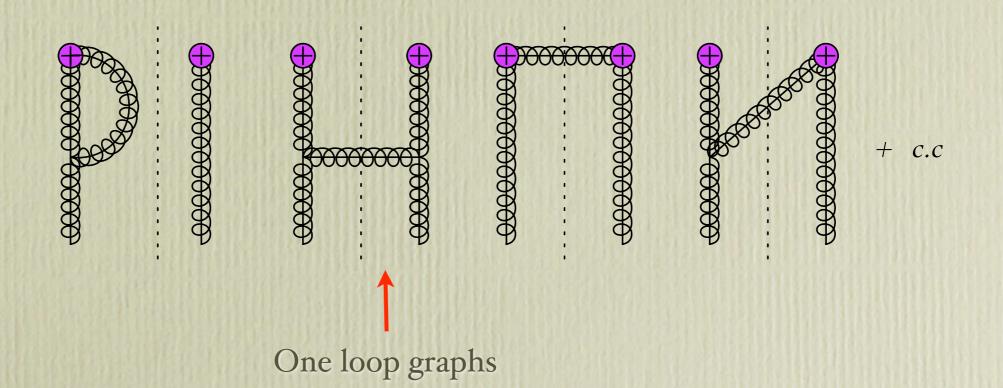
iBFs



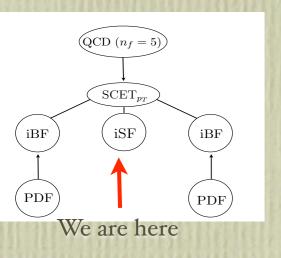
• Definition of the iBF:

We are here

$$\tilde{B}_{n}^{\alpha\beta}(x_{1}, t_{n}^{+}, b_{\perp}, \mu) = \int \frac{db^{-}}{4\pi} e^{\frac{i}{2}\frac{t_{n}^{+}b^{-}}{Q}} \sum_{\text{initial pols. } X_{n}} \sum_{X_{n}} \langle p_{1} | \left[gB_{1n\perp\beta}^{A}(b^{-}, b_{\perp}) | X_{n} \rangle \right] \times \langle X_{n} | \delta(\bar{\mathcal{P}} - x_{1}\bar{n} \cdot p_{1}) gB_{1n\perp\alpha}^{A}(0) \right] | p_{1} \rangle,$$

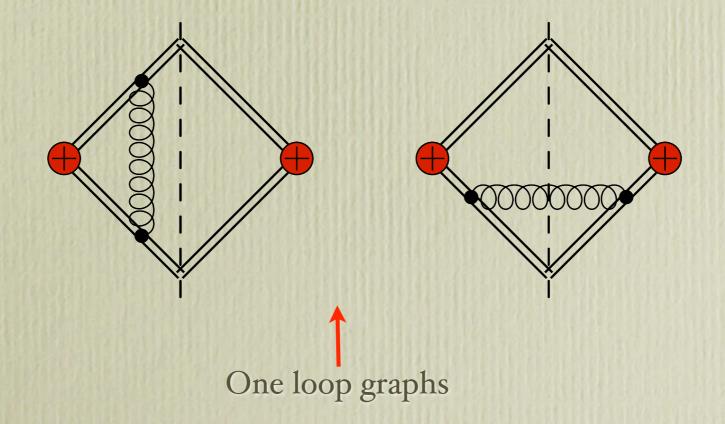


Soft function



• Soft function definition:

$$S(z) = \langle 0|\operatorname{Tr}(\bar{T}\{S_{\bar{n}}T^DS_{\bar{n}}^{\dagger}S_nT^CS_n^{\dagger}\})(z)\operatorname{Tr}(T\{S_nT^CS_n^{\dagger}S_{\bar{n}}T^DS_{\bar{n}}^{\dagger}\})(0)|0\rangle$$



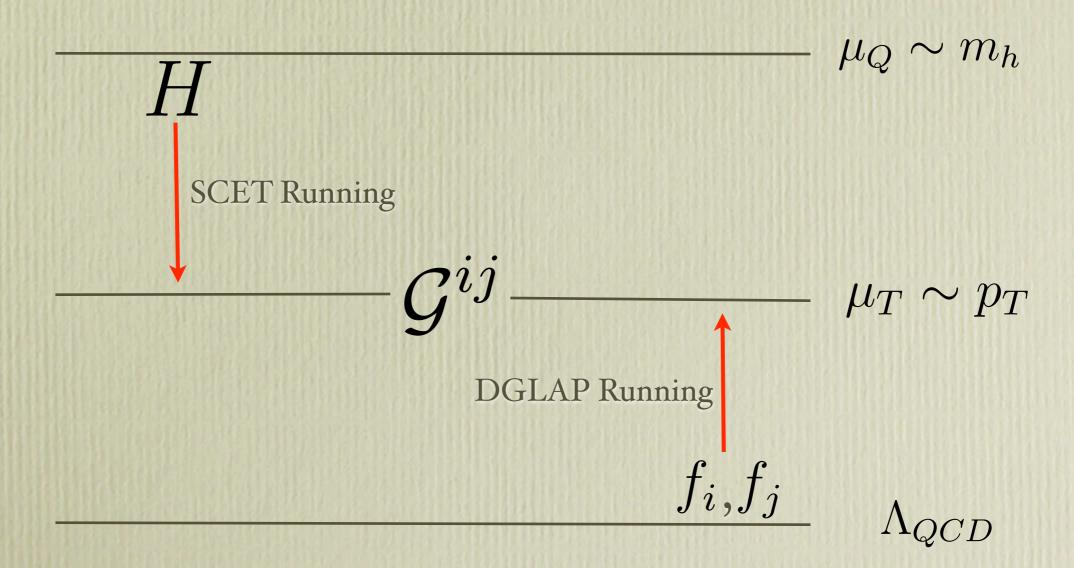
Running

Running

• Factorization formula:

$$\frac{d^2\sigma}{dp_T^2dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

• Schematic picture of running:



Running

• Factorization formula:

$$\frac{d^2\sigma}{dp_T^2dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

$$H = |C(\mu_Q, Q)|^2 \exp\left\{ \int_{\mu_T}^{\mu_Q} \frac{d\mu}{\mu} \Gamma_c \left[\alpha_s(\mu)\right] \ln\left(\frac{Q^2}{\mu^2}\right) + \gamma \left[\alpha_s(\mu)\right] \right\}$$

$$\Gamma_c \left[\alpha_s \right] = A_{CSS},$$

$$\gamma^{(1)} = B_{CSS}^{(1)},$$

$$\gamma^{(2)} = B_{CSS}^{(2)} + \text{pieces from C, } \mathcal{G}$$

Limit of very small pT

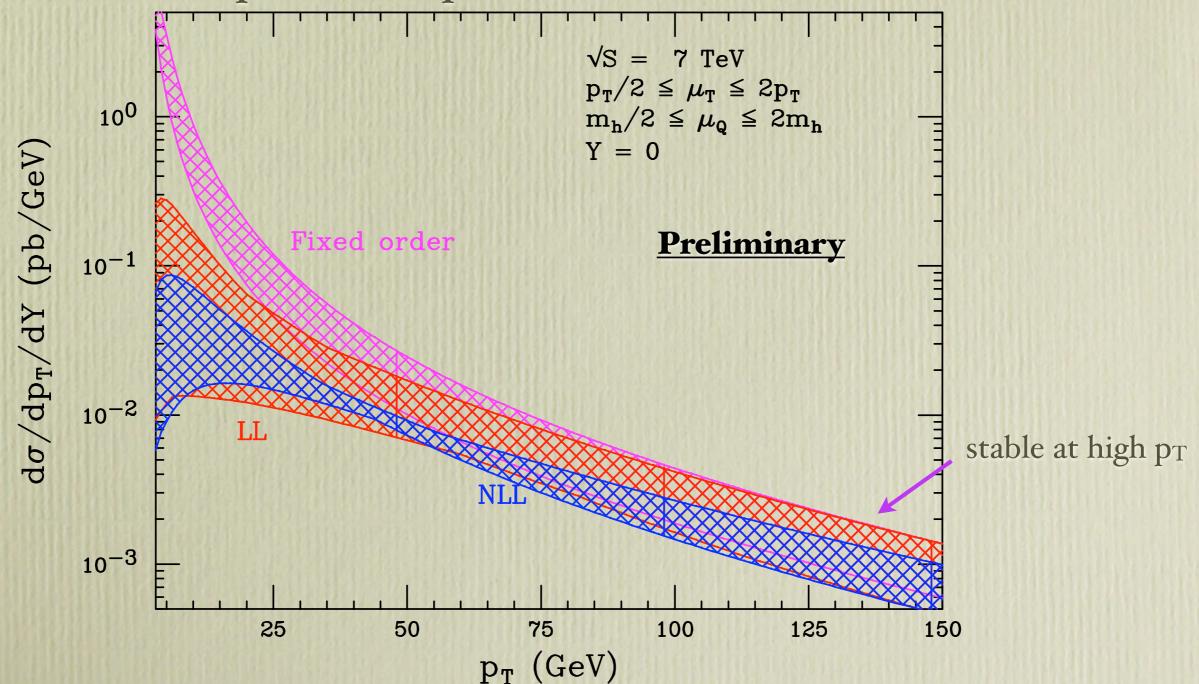
• We derived a factorization formula in the limit:

$$m_h \gg p_T \gg \Lambda_{QCD}$$

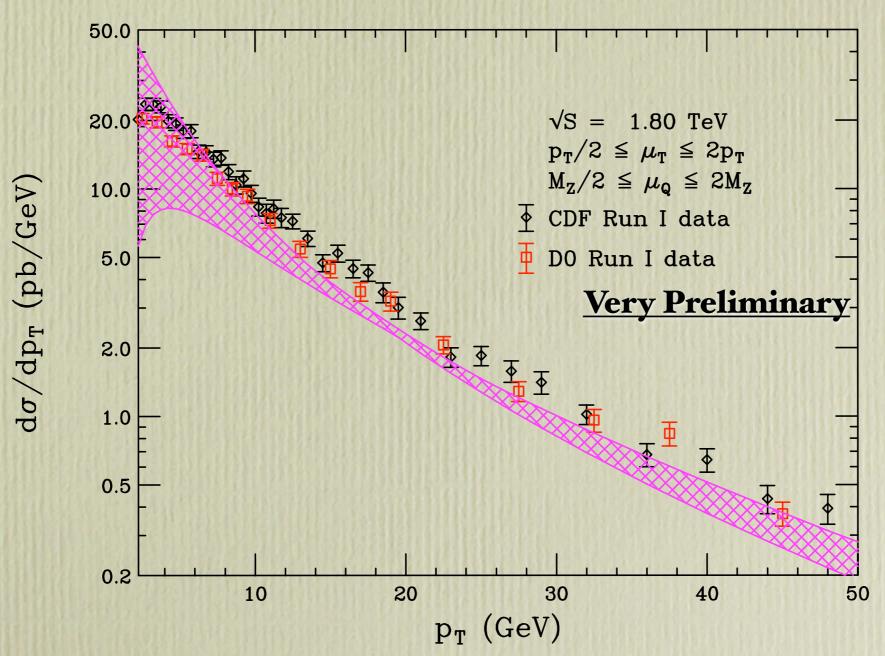
- For smaller values of pT, one can introduce a nonperturbative model for the transverse momentum function: field theoretically defined, running known
- In the limit $p_T=0$, $m_h \rightarrow \infty$, $d\sigma/dp_{T^2} \rightarrow constant$ Parisi, Petronzio
- Dominated by back-to-back hard jets⇒in SCET, this is a power-suppressed operator
- Leading term Sudakov suppressed in this limit
- Working to understand this in SCET...

Higgs at the LHC

 Matching accomplished just by subtracting expanded exponent from fixed order



Tevatron Z production



Missing 2-loop iBF, soft functions needed for full NNLL+NLO

Conclusions

• Derived factorization formula for the Higgs transverse momentum distribution in an EFT approach:

$$\frac{d^2\sigma}{dp_T^2dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Resummation via RG equations in EFTs
- Formulation is free of Landau poles; easy matching to fixed-order
- Next steps: higher-order calculations of iBF, iSF to enable NNLL+NLO result, modeling of low p_T